Trigonometric Functions

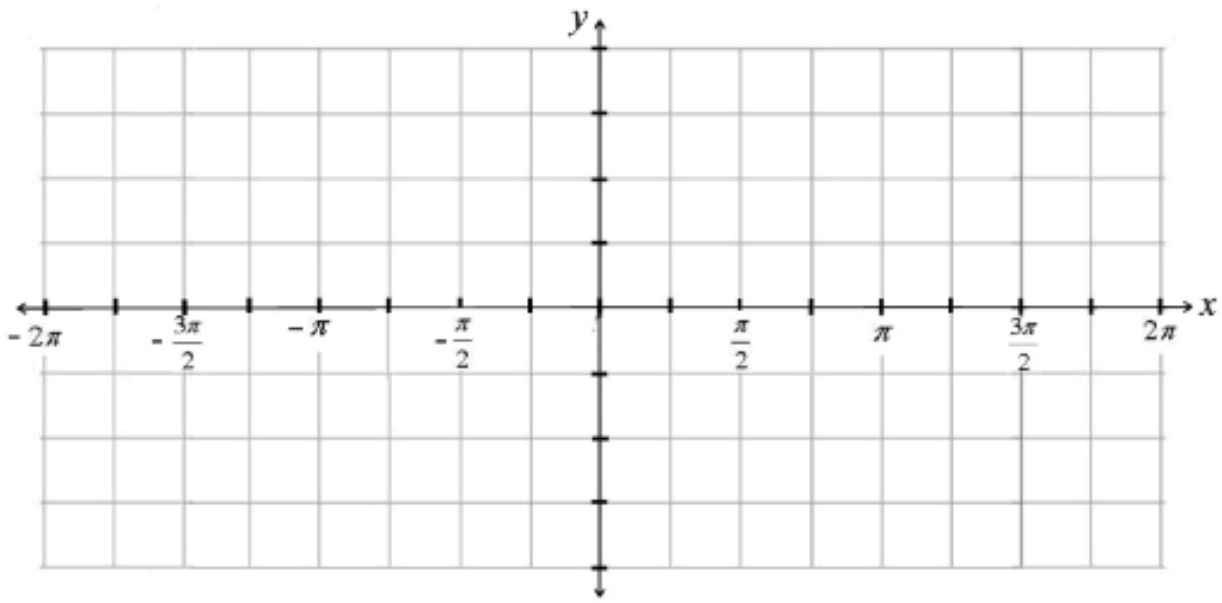
These notes are intended as a summary of section 6.5 (p. 513 – 520) in your workbook. You should also read the section for more complete explanations and additional examples.

Periodic Functions

A function that repeats its values in regular intervals over its domain is a **periodic function**. The sine, cosine, and tangent functions are examples of periodic functions. The length of the interval over which the values repeat (measured along the x-axis) is called the **period** of the function.

The Sine Function

Draw the graph of $y = \sin x$ on the axes below.



The sine function has the following properties:

Domain:

Period:

Zeros:

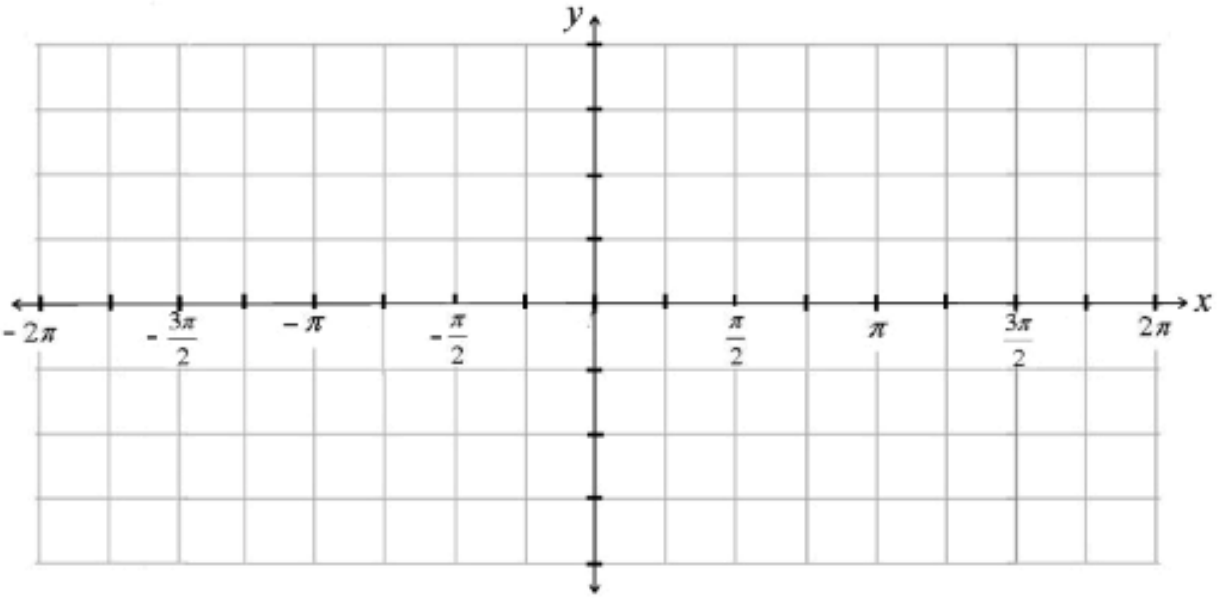
Maximum:

Minimum:

Range:

The Cosine Function

Draw the graph of $y = \cos x$ on the axes below.



The cosine function has the following properties:

Domain:

Period:

Zeros:

Maximum:

Minimum:

Range:

Functions whose graphs have the same shape as $y = \sin x$ or $y = \cos x$ are called **sinusoidal functions**. A sinusoidal function has a maximum and minimum value that are equidistant from the center line of the graph (a line that is halfway between the maximum and the minimum). The **amplitude** of a sinusoidal function is the distance from a maximum or a minimum to the center line.

All of the transformations that we learned in chapter 3 can be applied to graphs of trigonometric functions.

Determining Amplitude

The **amplitude** of a sinusoidal function is equal to $|a|$ when the function is written in the form:

$$y = a\sin x \qquad \text{or} \qquad y = a\cos x$$

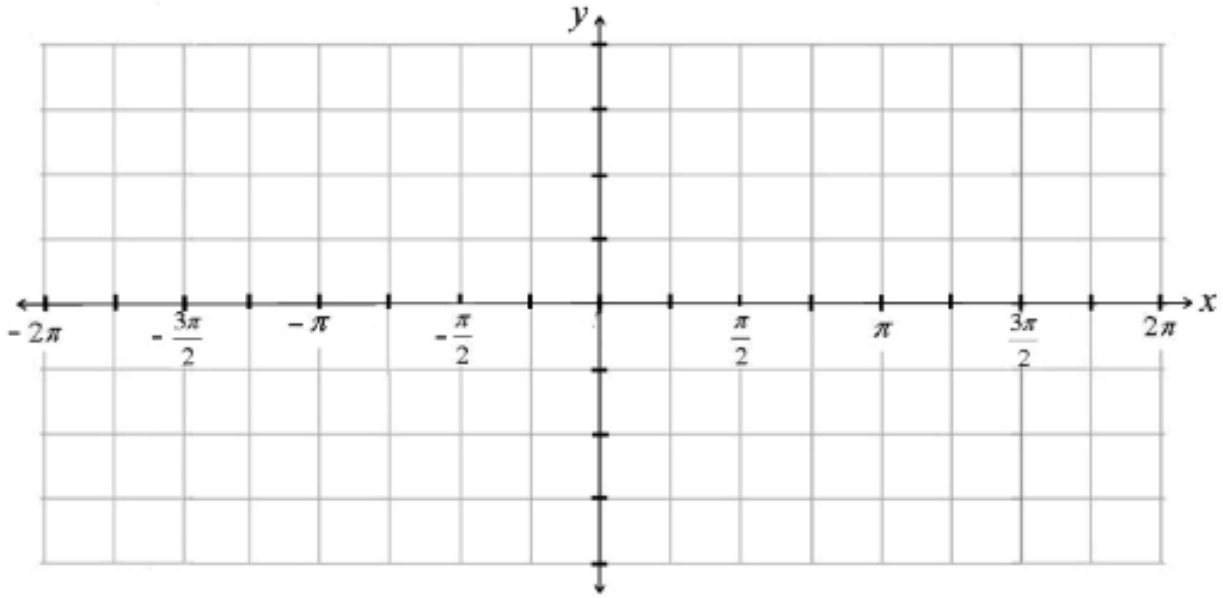
Example 1 (sidebar p. 516)
Determine the amplitude of the graph of each function.

a) $y = \frac{2}{3}\sin x$

b) $y = -4\cos x$

The Tangent Function

Draw the graph of $y = \tan x$ on the axes below.



The tangent function has the following properties:

Asymptotes:

Domain:

Period:

Zeros:

Range:

Determining Period

The **period** of $y = \sin bx$ or $y = \cos bx$ is $\frac{2\pi}{b}$. The period of $y = \tan bx$ is $\frac{\pi}{b}$.

Example 2 (sidebar p. 517)
Determine the period of each function.

a) $y = 6\cos x$

b) $y = \tan\frac{2}{3}x$

c) $y = \sin\frac{x}{7}$

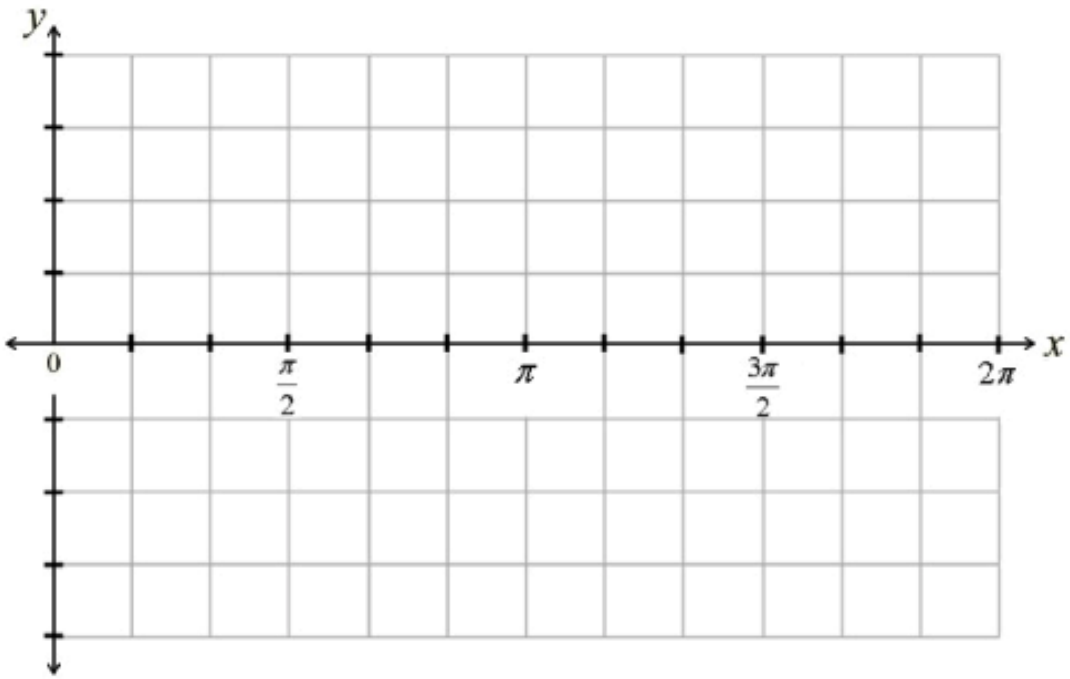
Determining Phase Shift

The graph of $y = \sin(x - c)$ is the image after the graph of $y = \sin x$ has been translated c units horizontally. This distance is called the **phase shift** of the function.

Example 3 (sidebar p. 518)

a) Determine the phase shift of the function $y = \cos\left(x - \frac{\pi}{6}\right)$.

b) Sketch graphs of $y = \cos x$ and $y = \cos\left(x - \frac{\pi}{6}\right)$ for $0 \le x \le 2\pi$.

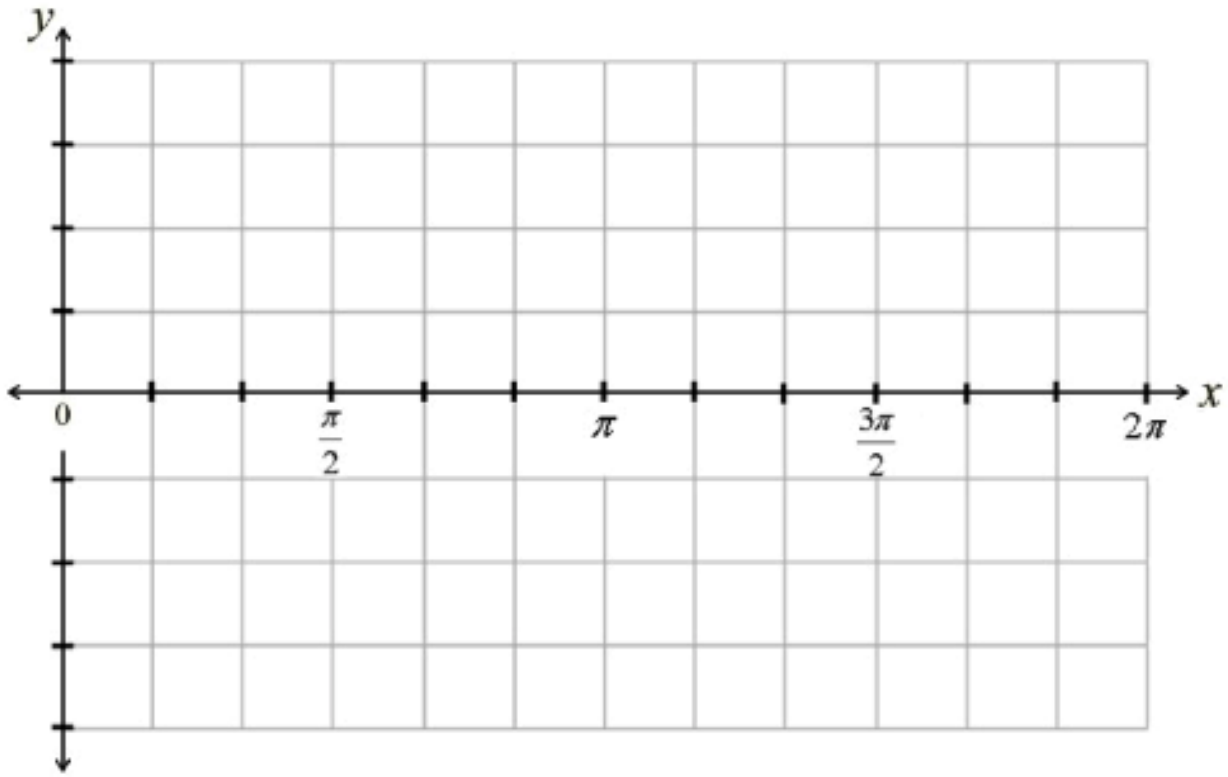


Example 4 (sidebar p. 519)
Describe how the graph of each function relates to the graph of $y = \sin x$. Then, on the same grid, sketch the graphs of $y = \sin x$ and each function below, for $0 \le x \le 2\pi$.

a) $y = 3\sin x$

b) $y = \sin 3x$

c) $y = \sin x + 3$



Homework: #3 – 7, 9 in the exercises (p. 521 – 526). Answers on p. 527.